South African Computer Olympiad
Web Training, 2009
IOI Squad May Contest

Overview

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Problem	**tile**	**destroy**	**jelly**	**tiles**
Source	tile.c tile.cpp	destroy.c destroy.cpp	jelly.c jelly.cpp	tiles.c tiles.cpp
Input file	stdin	stdin	stdin	stdin
Output file	stdout	stdout	stdout	stdout
Time limit	1 second	1 second	1 second	1 second
Number of tests	10	10	10	10
Points per test	10	10	10	10
Detailed feedback	No	Yes	No	No
Total points	**100**	**100**	**100**	**100**

The maximum total score is 400 points.

Tiling Mall

Francois Conradie

Introduction

A new mall is being constructed in Cape Town. The mall is nearly complete and all that is left to do is to tile the main corridor with 2×1 tiles. Mr. Creosote, the owner of the mall, is a workaholic lacking some common sense and insists on seeing every possible tiling of the main corridor. Hendrik, one of the smarter employees, immediately realized that this would be impossible — there are simply too many possibilities. Mr. Creosote, however, demands proof of this. Hendrik has asked you to help him determine in how many different ways you can tile the corridor.

Task

Given an infinite number of 2×1 tiles, your task is to help Hendrik determine in how many different ways you can tile a $4 \times N$ corridor using only these tiles.

Example

If you have a 4×2 corridor then there are 5 different ways of laying the tiles as illustrated in Figure 1.

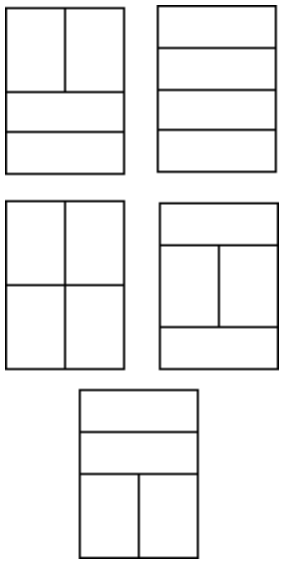

Figure 1: *All possible tilings of a* 4×2 *corridor.*

Input (stdin)

The first and only line contains a single integer N.

Sample input

2

Output (stdout)

Output the remainder when the total number of configurations is divided by 1 000 007.

Sample output

6

Constraints

- $1 \leq N \leq 5\,000\,000$

Additionally, in 50% of the test cases:

- $1 \leq N \leq 15$

Time limit

1 second.

Scoring

A correct solution will score 100% while an incorrect solution will score 0%.

Destroy the connection

Schalk-Willem Krüger

Introduction

A while ago Bruce and Carl commissioned you to test their proposed copy protection system of their software company called "Good and Evil Tech". However, they fired you, because you couldn't solve the "Travelling Salesman Problem" in constant time.

You are very furious and want to take revenge by breaking the communication between their computers and all the servers of the company.

Task

Bruce and Carl have several computers that may or may not be on the same network as all the central servers. The network consists of many computers with wires linking some pairs of those computers. There will be at most one wire in each direction between each pair of computers and there can be pairs of computers without a wire between them.

You can destroy computers and wires, but you cannot destroy Bruce or Carl's computers or their servers as Bruce made them indestructable. You have estimated the cost of blowing up each computer and the cost of cutting each wire in the network.

You want to determine the minimum cost of interrupting the communication between Bruce and Carl's computers and all the servers.

Example

Suppose there are 4 computers in the network. The first computer belongs to Bruce and is connected with wires of cutting cost 3 to the second and third computers. Computer two and three have costs of 1 and 5 to destroy respectively and are connected to the server (computer four) with wires of cost 2 and 4 respectively.

The minimum cost to disconnect Bruce's computer from the server is 4: destroy computer two (cost 1) and the wire connecting the first and third computer (cost 3).

Input (stdin)

- The first line of input contains two space-separated integers: M and W, representing the number of computers (including Bruce and Carl's computers and the servers) and the number of wires in the network respectively.
- The second line contains two space-separated integers: T and S, representing the number of computers owned by Bruce and Carl and the number of servers.
- The next $M - T - S$ lines contain the cost of blowing up computer $B + 1$ to computer $T - 1$. The first T computers belong to Bruce and Carl and the last M computers are the servers. Bruce and Carl's computers and the servers cannot be destroyed and their costs are thus not given. Each line contains a single integer: the cost, C_i, of destroying computer i and with i going from $B + 1$ up to $T - 1$.
- The last W lines each contain three space-separated integers, X_i, Y_i and D_i, indicating that the wire that connects computer X_i to computer Y_i costs D_i. Each pair of computers can have at most two wires (one in each direction).

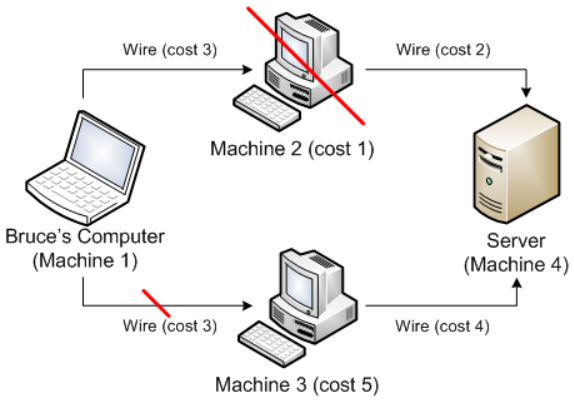

Figure 2: *An image illustrating the sample input.*

Sample input

```
4 4
1 1
1
5
1 2 3
1 3 3
2 4 2
3 4 4
```

Output (stdout)

Output a single integer: the minimum cost of interrupting the communication between the computers of Bruce and Carl and all the servers.

Sample output

```
4
```

Constraints

- $2 \leq M \leq 400$
- $0 \leq W \leq 70\,000$
- $1 \leq T \leq M - 1$
- $1 \leq S \leq M - 1$
- $T < i < M - S$
- $0 \leq C_i, D_i \leq 100\,000$
- $C_i <> D_i$
- $1 \leq X_i, Y_i \leq M$

Additionally, in 30% of the test cases:

- $2 \leq M \leq 60$
- $0 \leq W \leq 4\,000$

Time limit

1 second.

Detailed feedback

Detailed feedback is enabled for this problem.

Scoring

A correct solution will score 100% while an incorrect solution will score 0%.

n

Jelly Tots

Graham Manuell

Introduction

It is Bruce's mother's birthday, but with Bruce having spent all his money on world peace, he does not have any left to buy her a present, so he has decided to make one. He plans to stack coloured jelly tots into a plastic tube in an interesting design. He carefully divides the jelly tots into piles of different colours. Before long he finds himself thinking about how many patterns he could possibly create. He notes that because the tube is symmetrical, one design and its reverse should only be counted once. In order to find the solution he uses Burnside's lemma to show that the answer is the average of the total number of designs and the palindromic designs. Bruce easily comes up with the final answer, but alas, while his mind was drifting, he left the jelly tots in the sun and some of them have fused together. The jelly tots are now all in segments of 2 or 3. Each segment is made up of jelly tots of only one colour.

Task

Help Bruce find the new number of possible designs (with and without considering symmetry) if the tube can hold N single jelly tots and he has an infinite supply of segments in C different colours. He only cares about the colour of the design, so even if he uses different combinations of segments for two arrangements, if the jelly tots in corresponding positions have the same colour, the arrangements are the same. Because there are potentially very many designs, you should give the answers modulo 100 000 007.

Example

If Bruce has two colours of jelly tots, red (R) and blue (B), he can make 10 different designs of length 6: RRRRRR, RRRRBB, RRRBBB, RRBBRR, RRBBBB, BBRRRR, BBRRBB, BBBRRR, BBBBRR, BBBBBB. Note that adding 2 red 3-jelly-tot segments and adding 3 red 2-jelly-tot segments would look that same and hence count as the same design. Out of these RRRRRR, RRBBRR, BBRRBB, BBBBBB are palindromic, so when considering symmetry the answer is $(10 + 4)/2 = 7$, which can be seen to be correct.

Input (stdin)

The input consists of a single line containing two space-separated integers, N and C.

Sample input

```
6 2
```

Output (stdout)

Output two space-separated integers, the number of designs modulo 100 000 007 not considering and then considering symmetry.

Sample output

```
10 7
```

Constraints

- $1 \leq N \leq 10\,000\,000$
- $1 \leq C \leq 10$

Additionally, in 30% of the test cases:

- $1 \leq N \leq 15$
- $1 \leq C \leq 4$

Time limit

1 second.

Scoring

A completely correct solution will score 100%. If only the number of designs without considering symmetry is correct 50% will be awarded, otherwise the solution will score 0%.

Tiles

Charl du Plessis

Introduction

Fred, the manic storekeeper, wants to re-tile his store. His store is an N (vertical) by M (horizontal) rectangle which will be covered by 1 by N tiles. He wants to tile the store such that the number of vertically placed tiles is equal to the number of horizontally placed tiles. Unfortunately, Fred has already started placing some of the tiles which we will term pre-tiles they cannot be taken away or moved.

Task

Given N and M and the positions of the pre-tiles, output the number of ways to tile the remainder of the rectangle such that in the entire N by M rectangle the number of vertical tiles equals the number of horizontal tiles. Let us number the blocks of the length of the rectangle. $1, 2, \cdots, M$. The position of a vertical pre-tile will be given by a number from 1 to M and the start (first block) of a horizontal pre-tile will also be given by a number from 1 to M. Furthermore, every N by 20 rectangle will contain at least one pre-tiled block. But since the number of ways to tile the store could get very large, output it modulo 1000000.

Example

Suppose $N = 4$ and $M = 16$. With vertical pre-tiles at 1 and 2; and 2 horizontal pre-tiles at 4. Then there exist 6 ways to tile the rectangle satisfying the conditions.

Input (stdin)

The first line will contain the space-seperated integers N and M. The next line will contain space-seperated integers V (number of vertical pre-tiles) and H (number of horizontal pre-tiles). The next V lines will each contain a single integer showing the position of a vertical pre-tile. The next H lines will contain a single integer showing the position of a horizontal pre-tile.

Sample input

```
4 16
2 2
1
2
4
4
```

Output (stdout)

A single integer: the number of ways that the store can be tiled satisfying the conditions, K, modulo 1000000.

Sample output

```
6
```

Constraints

- $2 \leq N \leq 10$
- $1 \leq M \leq 150$

Additionally, in 50% of the test cases:

- $K \leq 1000000$

Time limit

1 second.

Scoring

A correct solution will score 100% while an incorrect solution will score 0%.