Combinatorics

Permutations (n pick k)

With repetitions

$$n^k$$

Without repetitions

$$P(n,k) = \frac{n!}{k!(n-k)!}$$

Combinations (n choose k)

Without repetitions

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{n}{k+1} = \binom{n}{k}\frac{n-k}{k+1} \quad \text{where} \quad \binom{n}{0} = 1$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \quad \text{where} \quad \binom{n}{k} = 0 \text{ for } k > n$$

With repetitions
also number of ways to divide k identical objects into n sets

$$\left(\!\binom{n}{k}\!\right) = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$$

Multinomial Coefficients

Place n objects in m boxes of size k_i
Permutations of multiset with m distinct elements occurring k_i times

$$\binom{n}{k_1, k_2, k_3, \ldots, k_m} = \frac{n!}{k_1! k_2! k_3! \cdots k_m!} \quad \text{Note: } \sum_{i=1}^{m} k_i = n$$

Stirling Numbers of the second kind

The number of ways to partition a set of n elements into k non-empty subsets

$$S(n,k) = S_n^{(k)} = \left\{ {n \atop k} \right\}.$$

$$\left\{ {n \atop k} \right\} = \left\{ {n-1 \atop k-1} \right\} + k \left\{ {n-1 \atop k} \right\} \quad \text{where} \quad \left\{ {n \atop 1} \right\} = 1 \quad \text{and} \quad \left\{ {n \atop n} \right\} = 1.$$

Inclusion-exclusion principal

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{n} |A_i| - \sum_{i,j\,:\,1 \le i < j \le n} |A_i \cap A_j| + \sum_{i,j,k\,:\,1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| - \ \cdots \ + (-1)^{n-1} |A_1 \cap \cdots \cap A_n|$$

Burnside's Lemma

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

$|X/G|$ is the number of solutions taking symetry into account

G is the set fo transforms

$|X^g|$ is the number of solutions left unchanged by transform g

Calculating combinations

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function combination( n, k )

    c = 1

    for i = 0 to k-1

        c = c * (n-i)/(i+1)

    return c
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