Network Flow

Maximum Flow

Minimum Cut

What is Network Flow?

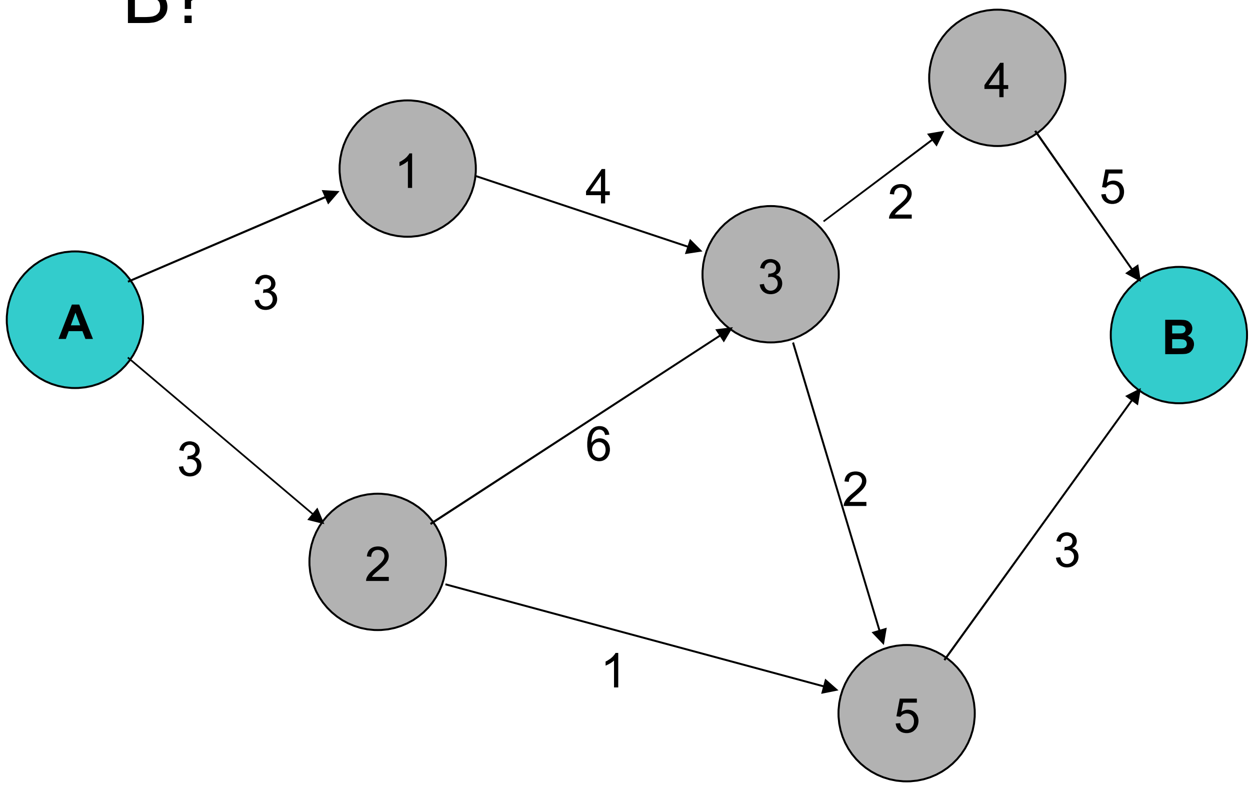
- Graph Theory
- Flow
- Direction
- A -> B
- e.g. Water flowing through pipes

Concepts and Definitions

- “Source” – Node in the graph emitting the “flow”
- “Sink” – Node in the graph consuming the “flow”
- “Capacity” – the maximum amount of “flow” that can pass through this edge/node.
- “Flow” – the amount of material passing through this node/edge/graph.

Max Flow Problem

- What is the maximum flow from A to B?



Ford-Fulkerson

- Simple
  - Initialize all flow counts to 0.
  - While there is an unsaturated path from source to sink:
    - Find the minimum capacity of all edges in that path
    - Increment the flow count of each edge in that path by the minimum
    - Increment the global flow count by that minimum
  - The total amount of flow in the global flow count is then maximum

Storage

- For each edge store:
  - Capacity
  - Current flow
- Undirected
  - Depends on problem for optimal storage
  - Store 2 flows (positive and negative) or sometimes 2 capacities.
  - Depending on problem, positive may cancel out negative and vice versa.

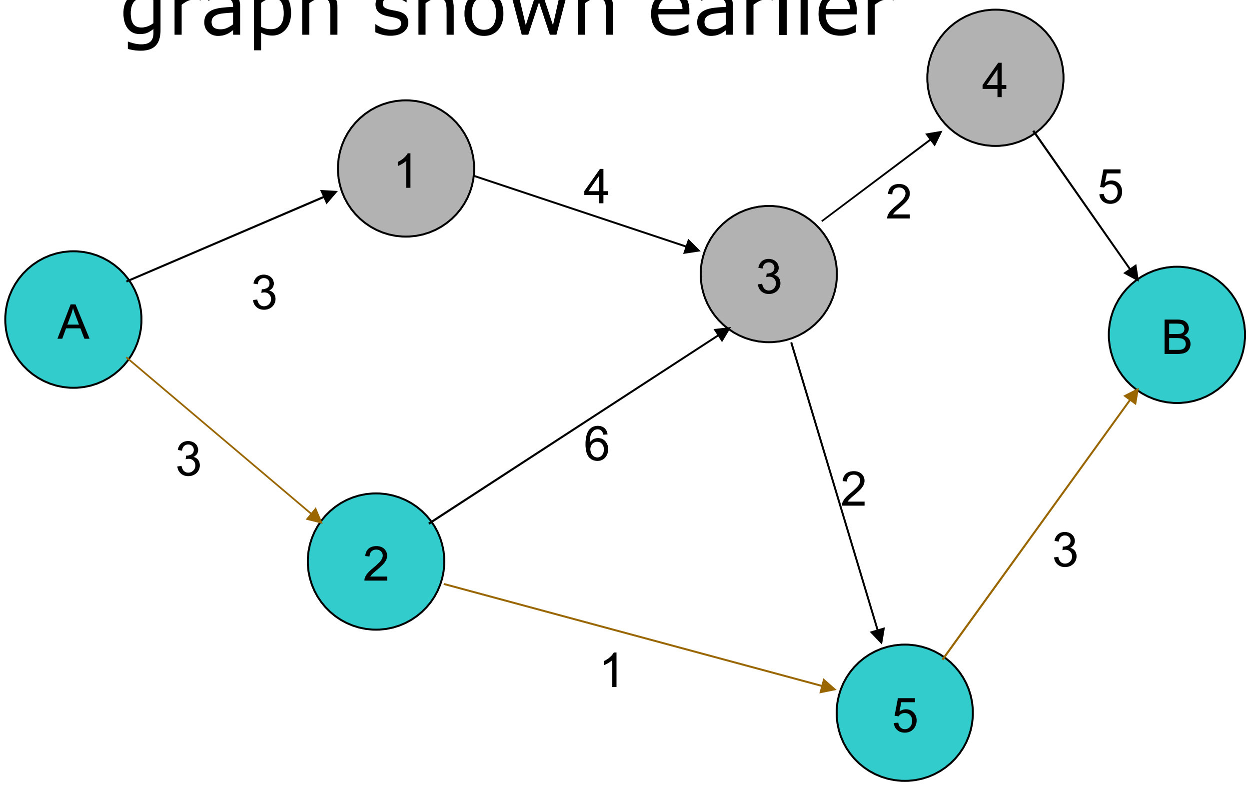
Finding the path

- Method does not affect answer
- Affects running time

- Good choices (depending on problem)
  - Shortest path (BFS)
  - Path with maximum flow
  - DFS - simplest

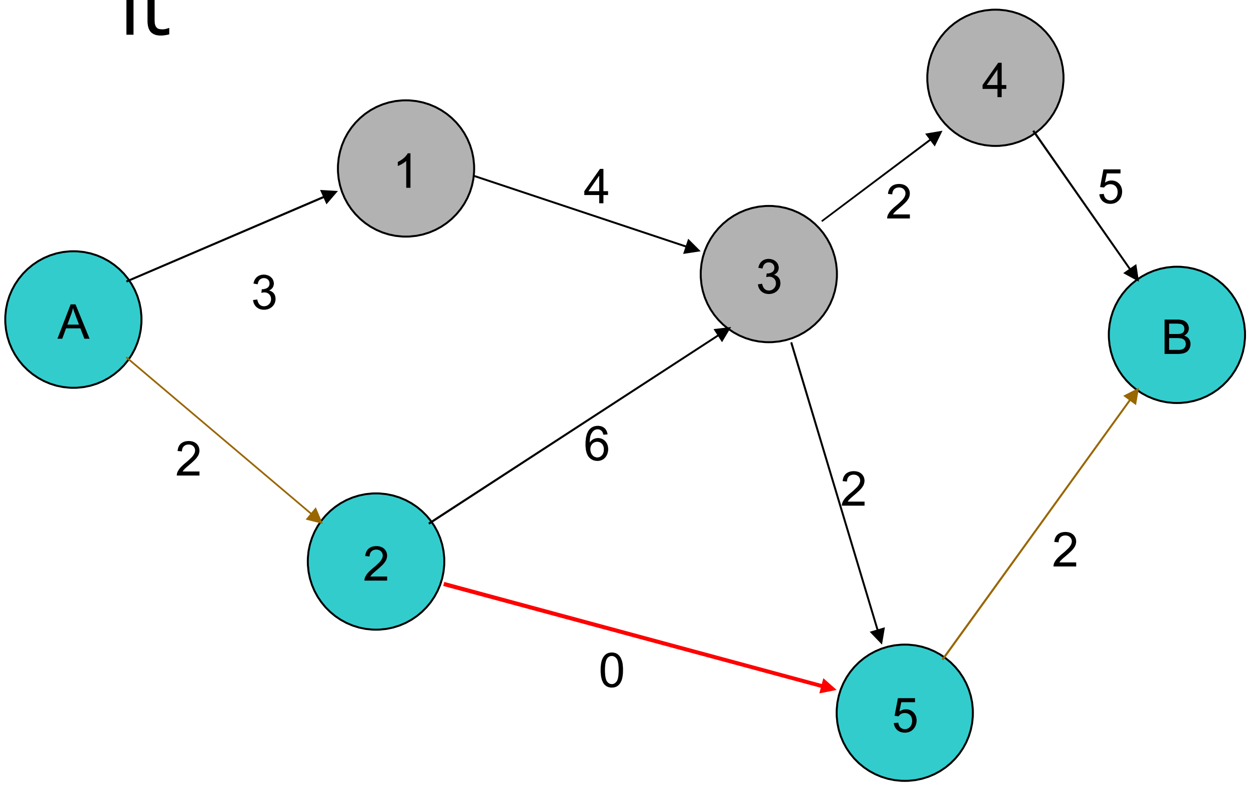
An example

- Determining maximal flow in the graph shown earlier



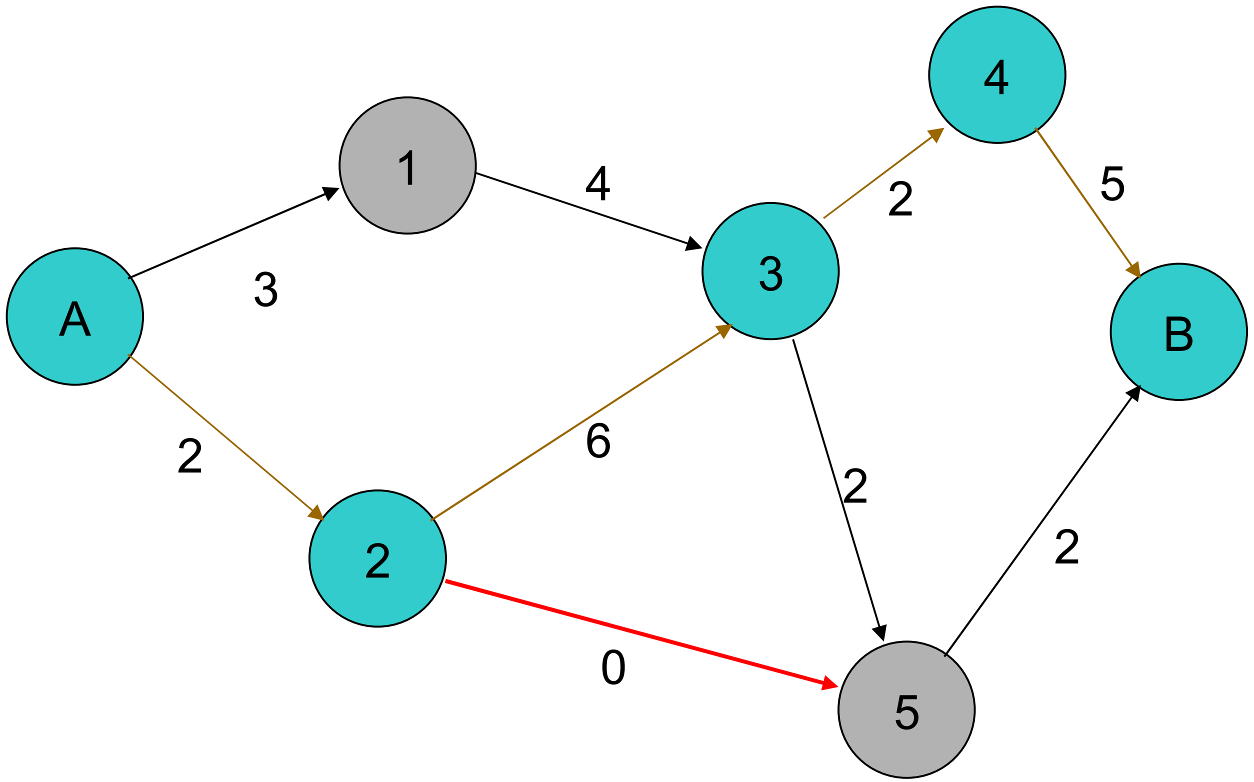
An example

- Choose a path and put flow through it



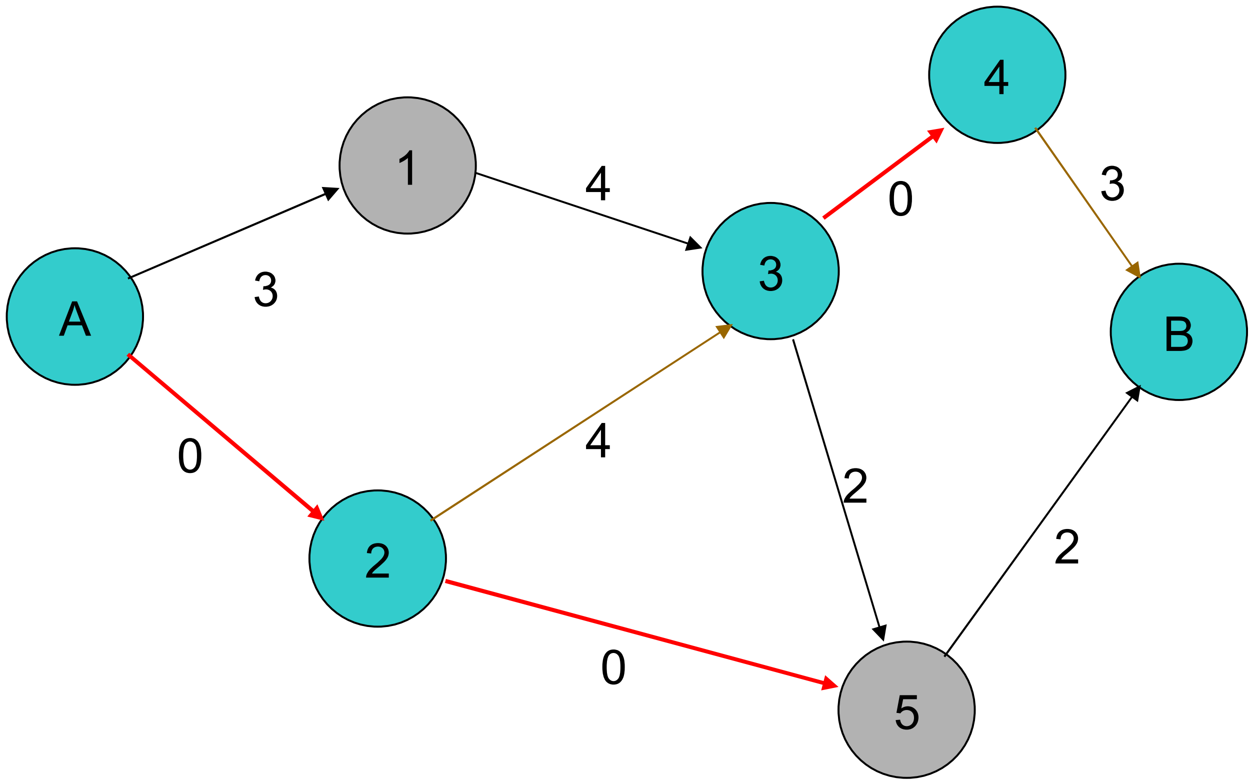
An example

- Current flow = 1



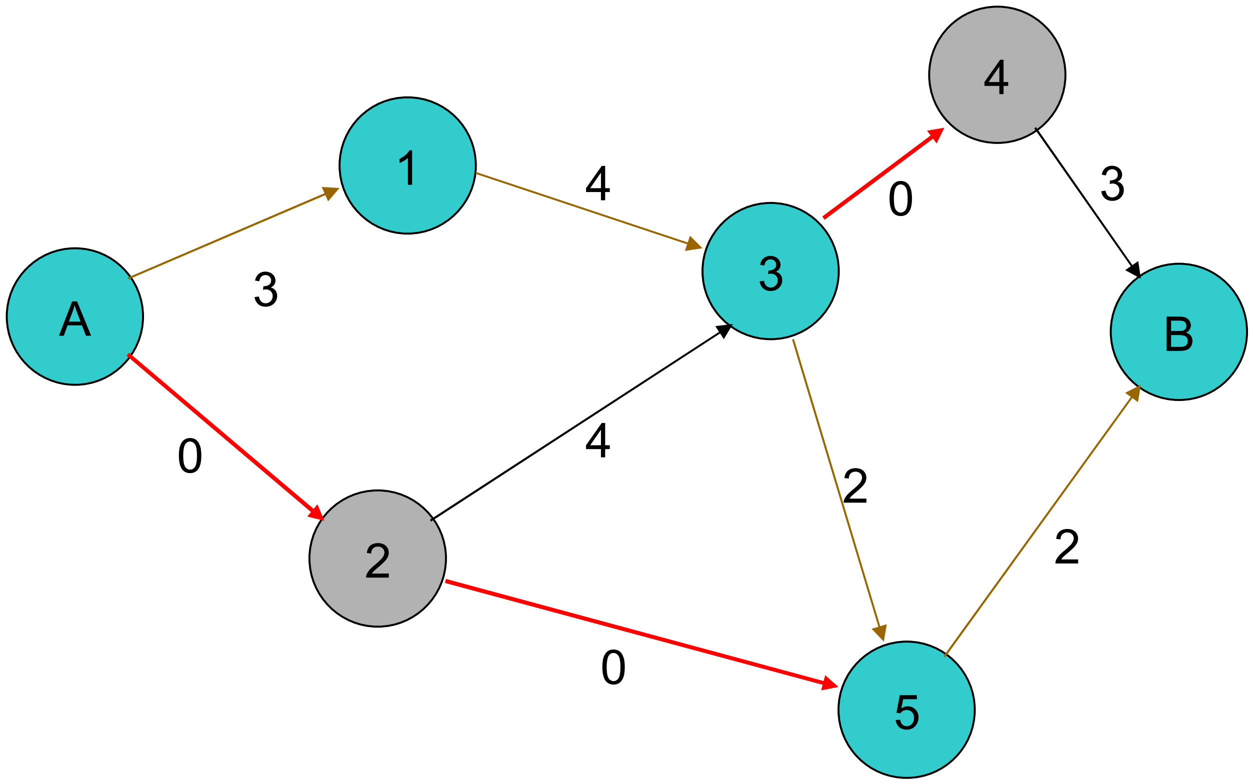
An example

- Current flow = 3



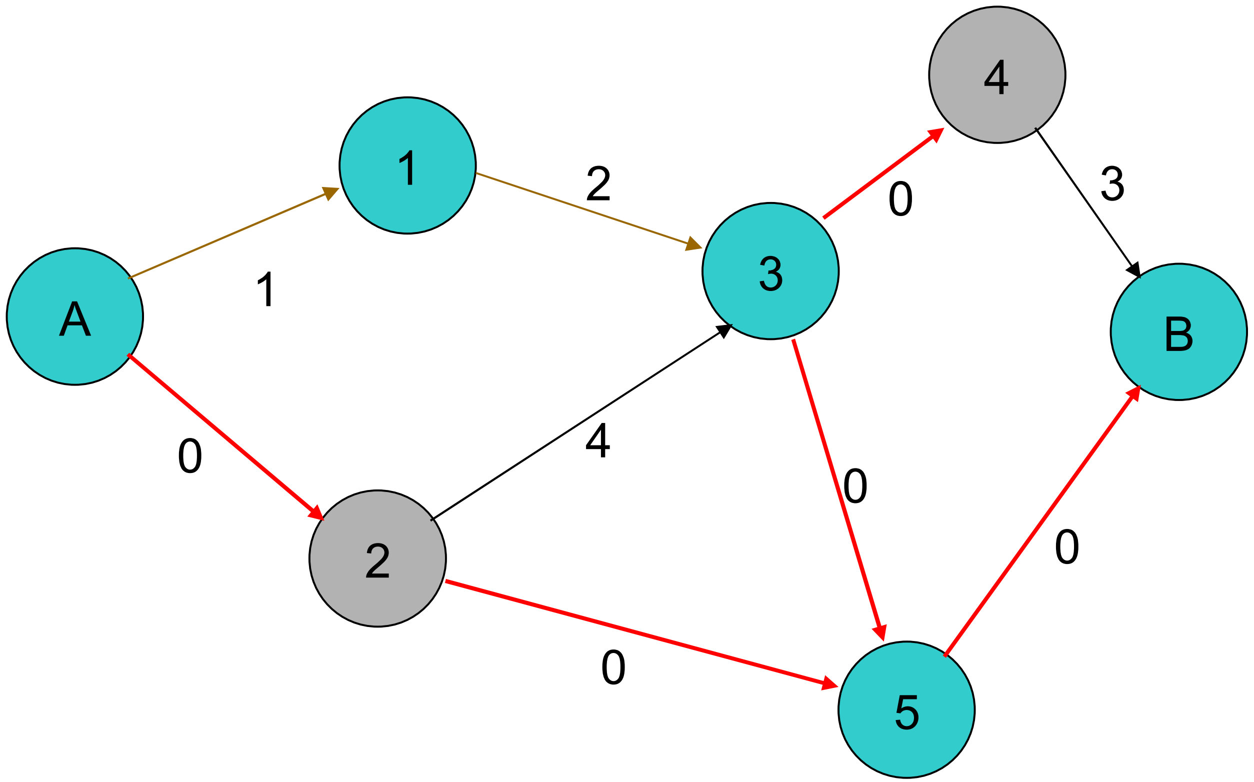
An example

- Current flow = 3



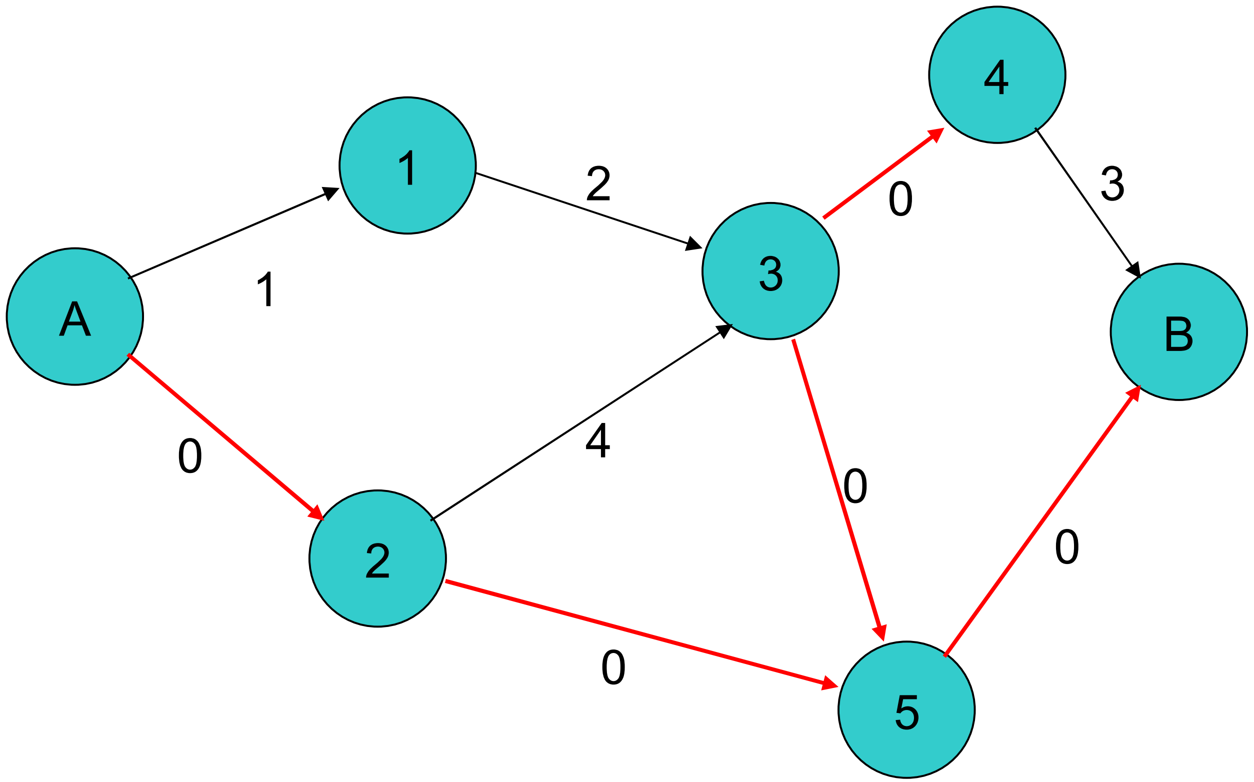
An example

- Current flow = 5



An example

- Total flow = 5

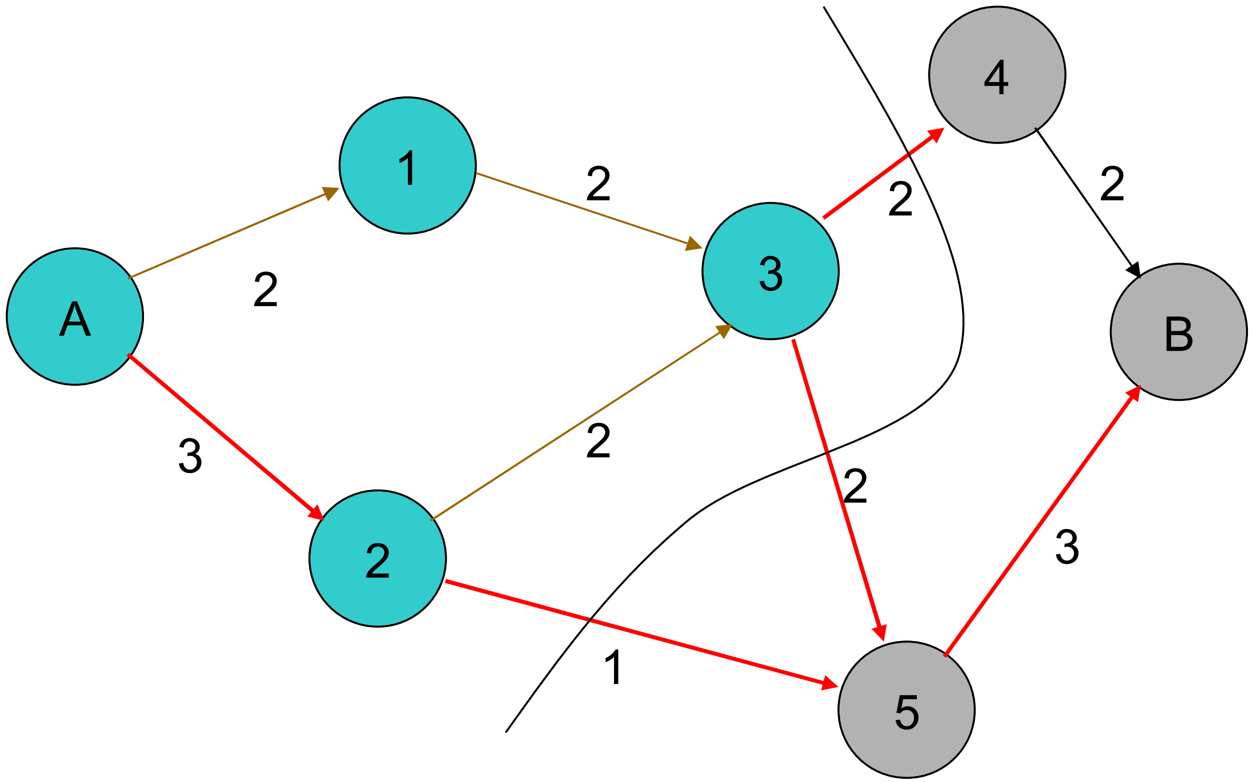


Minimum Cuts

- Maximum flow problems are tied in with Minimum Cut problems
- Minimum Cut
  - The minimum weighting of edges that separates two nodes (in this case source and sink)
- Sum of the weighting of cut edges = maximum flow through the network

An example

- Minimal cut = 1 + 2 + 2 = 5

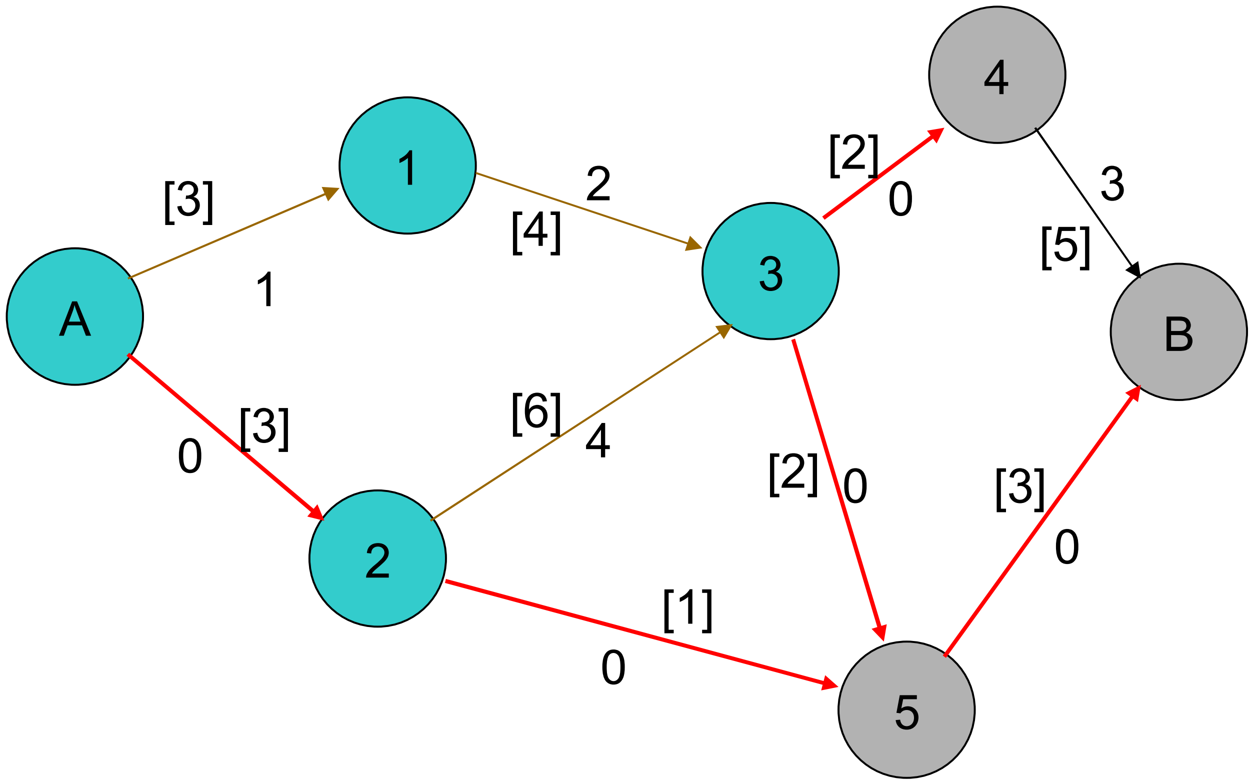


To find the minimum cut

- Create the maximum flow graph
- Select all nodes that can be reached from the source by unsaturated edges
- Cut all the edges that connect these nodes to the rest of the nodes in the graph
- This cut will be minimal

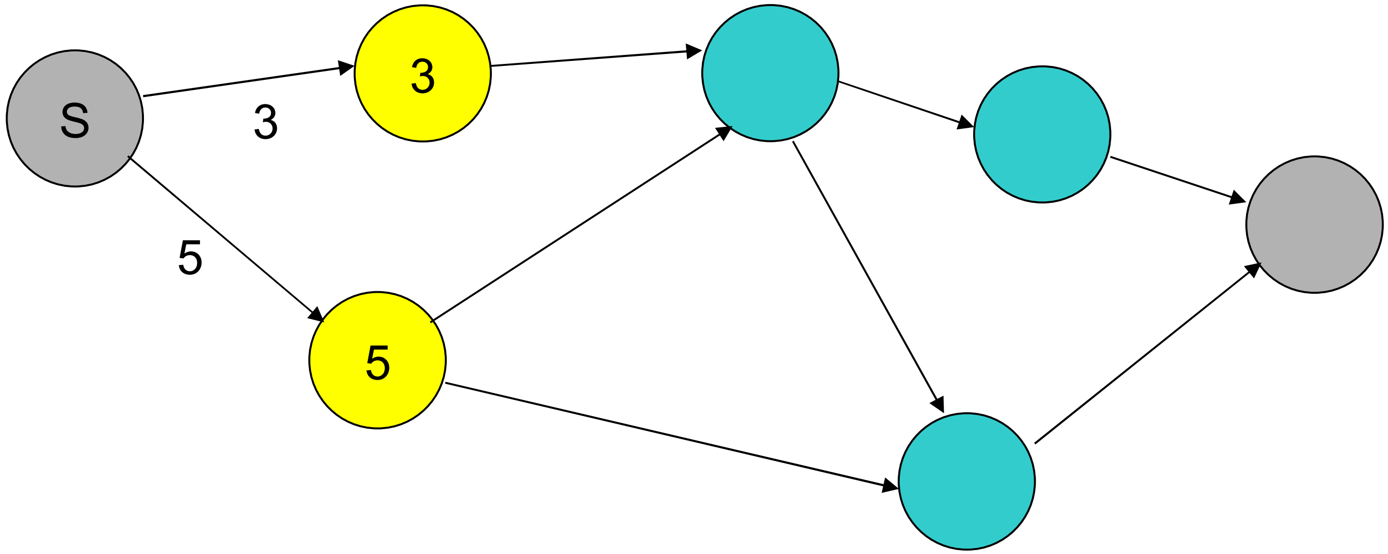
An example

- To find a minimal cut



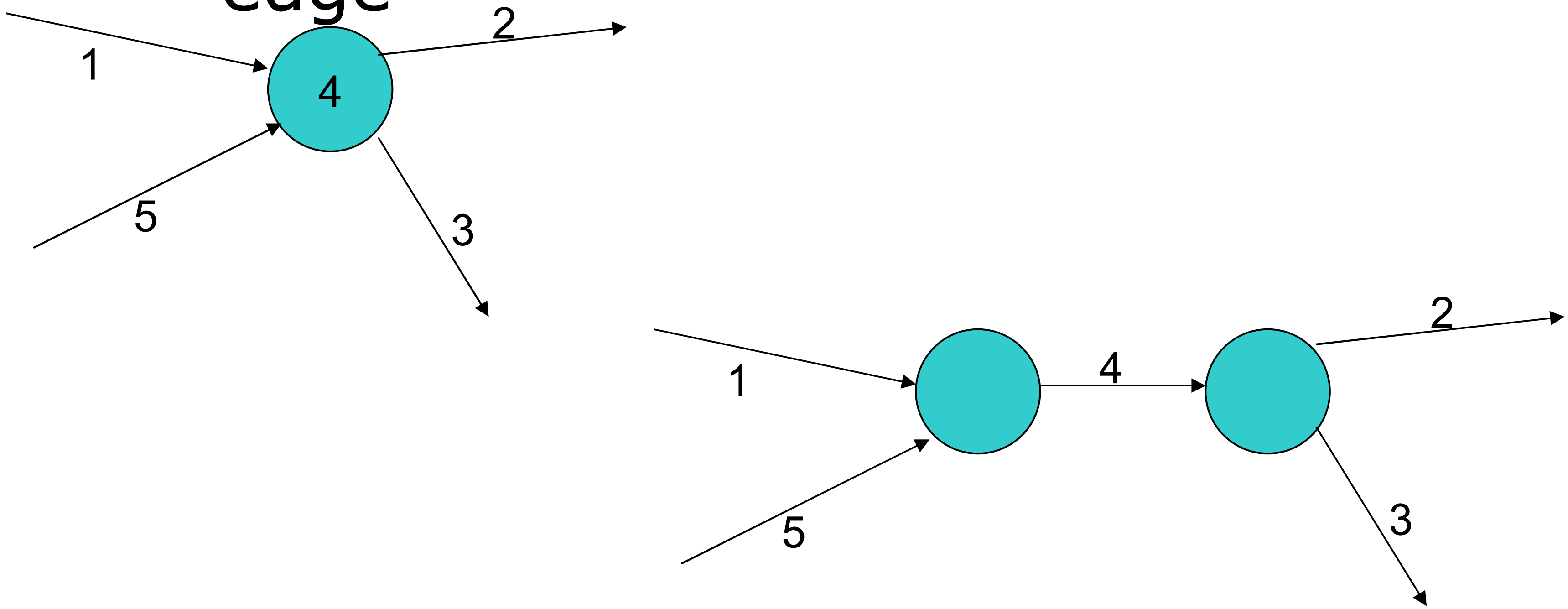
Variations on Network Flow

- Multiple sources & sinks – create a "supersource" or "supersink" which connects directly to each of these nodes, and has infinite capacity



Variations on Network Flow

- Node capacity – split the node into an “in” node, an “out” node and an edge



Conclusion

- Network flow problems
  - Graph Theory
  - Transporting some material from A to B
  - Along pathways (edges) that have capacities
  - Maximum flow is that maximum amount of material that can be transported from A to B.