Computational Geometry

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February 7, 2018

Vectors

Vectors

- a vector is a quantity with direction and magnitude
- a vector lists directional components

Representing Vectors

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$$

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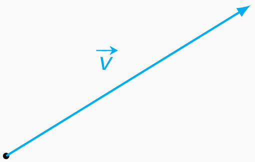

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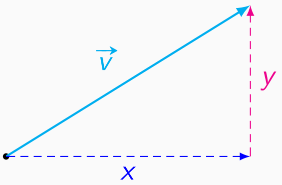

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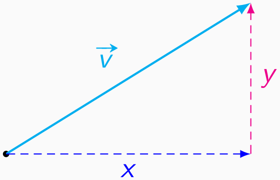


```
struct vec {
  int x, y;
};
vec v = {x, y};
```

Manipulating Vectors

Magnitude

$$|\vec{v}|$$
$$= \sqrt{x^2 + y^2}$$

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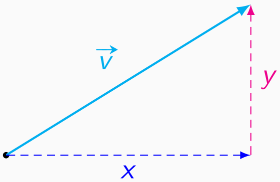

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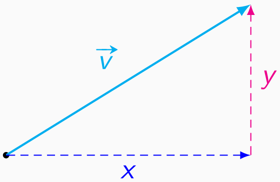


```
int len() {
  return sqrt(x*x + y*y);
}
```

Vector Addition

$$\vec{v}_1 + \vec{v}_2 = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$$

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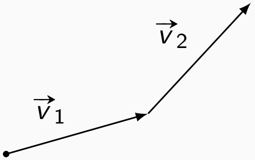

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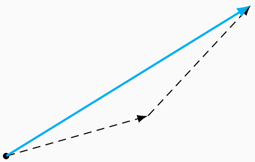

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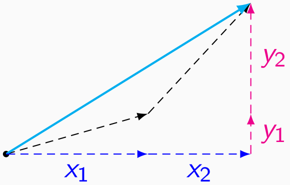

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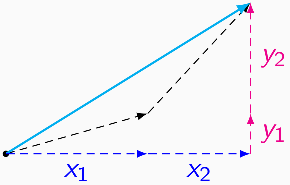


```
vec add(vec v) {
  return {x+v.x, y+v.y};
}
```

Scalar Multiplication

$$s\vec{v} = \begin{bmatrix} sx \\ sy \end{bmatrix}$$

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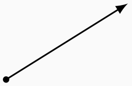

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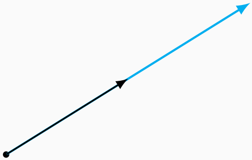

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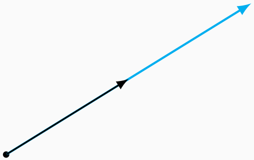

```
vec scale(int s) {
  return vec{s*x, s*y};
}
```

Dot Product

$$\vec{v}_1 \cdot \vec{v}_2$$
$$= |\vec{v}_1|\,|\vec{v}_2| \cos\theta$$
$$= x_1 x_2 + y_1 y_2$$

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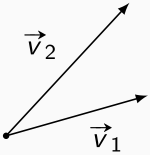

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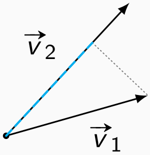

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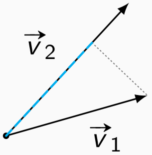


```
int dot(vec v) {
  return x*v.x + y*v.x;
}
```

Dot Product Uses

the dot product describes direction

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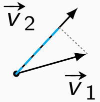
- $\vec{v}_1 \cdot \vec{v}_2 > 0$: $\vec{v}_1$ and $\vec{v}_2$ are directed towards the same half



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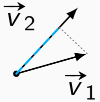

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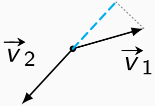

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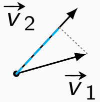

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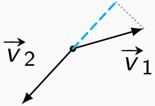
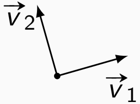


- $\vec{v}_1 \cdot \vec{v}_2 = 0$: $\vec{v}_1$ and $\vec{v}_2$ are perpendicular



Cross Product

$$
\begin{aligned}
&\vec{v}_1 \times \vec{v}_2 \\
&= |\vec{v}_1|\,|\vec{v}_2| \sin\theta \\
&= x_1 y_2 - y_1 x_2
\end{aligned}
$$

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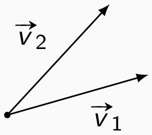

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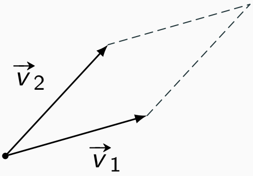

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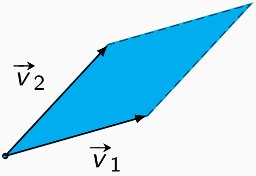

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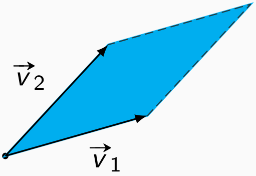

```
int cross(vec v) {
  return x*v.y - y*v.x;
}
```

Cross Product Uses

the cross product describes orientation

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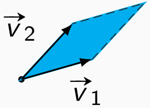
- $\vec{v}_1 \times \vec{v}_2 > 0$: $\vec{v}_1$ is clockwise from $\vec{v}_2$



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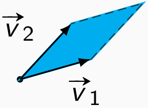

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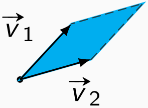

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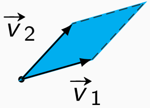

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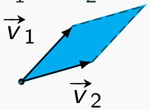

- $\vec{v}_1 \times \vec{v}_2 = 0$: $\vec{v}_1$ and $\vec{v}_2$ are collinear

Lines

Lines

lines can be represented as a pair of vectors

- $\vec{p} = \vec{b} + s\vec{m}$
- $\vec{b}$: a vector for position
- $\vec{m}$: a vector for slope
- the vectors must be standardised in order to check for equality

Representing Line Segments

$$\overline{AB}$$

- a line segment can be represented by a pair of vectors
- $\vec{p}_1 - \vec{p}_0$ gives a vector representing the line segments' length and direction

Intersecting Line Segments

the cross product allows efficient checks for intersecting line segments

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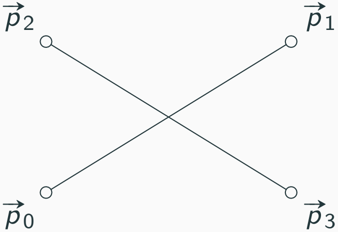

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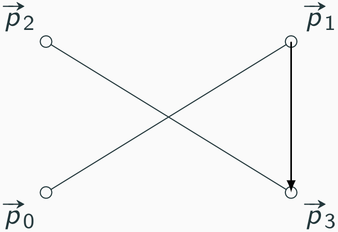

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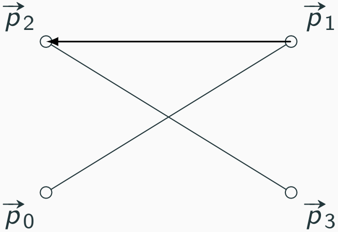

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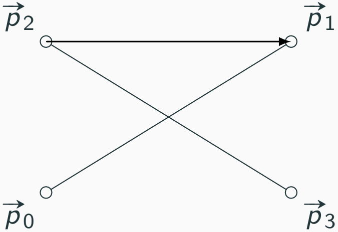

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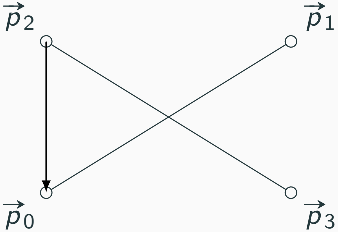

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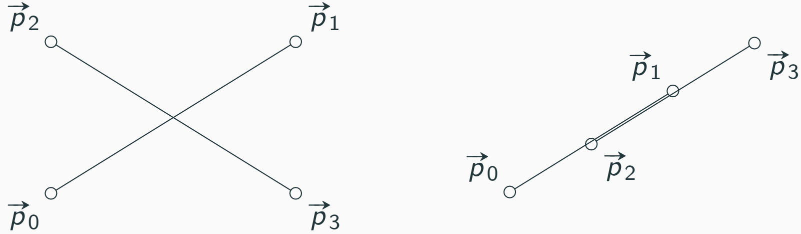
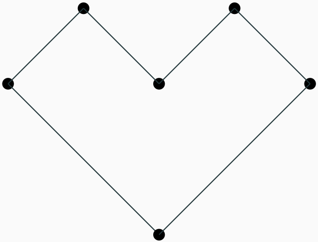


Polygons

calculating the area of a polygon can be simplified by triangulating the polygon



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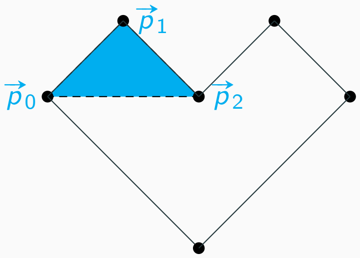

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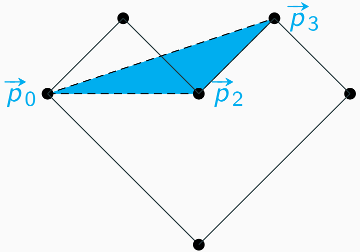

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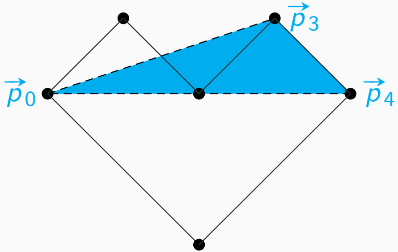

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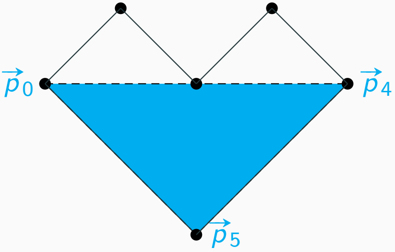

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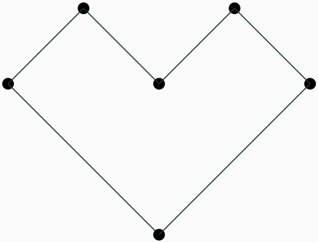
the cross product is used to determine the area



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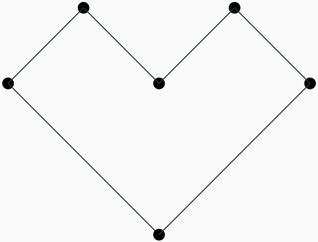

the cross product is used to determine the area

- it returns the area of the parallelogram with two vectors common to each triangle

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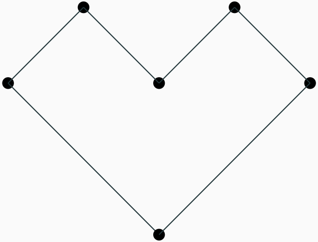

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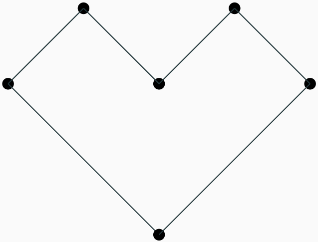

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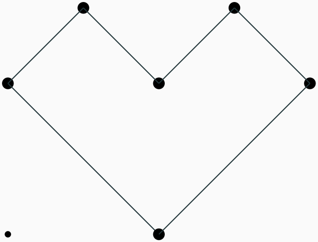
$$2A(\Omega) = \left| \sum_{i=1}^{n-2} (\vec{p}_i - \vec{p}_0) \times (\vec{p}_{i+1} - \vec{p}_0) \right|$$

Optimising Area of Polygons



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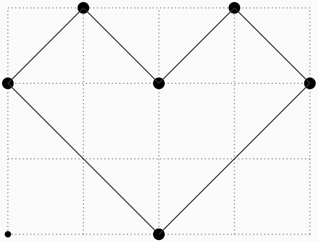

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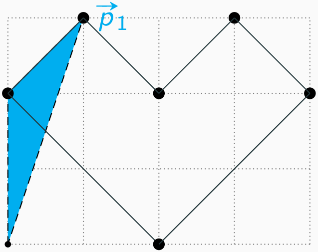

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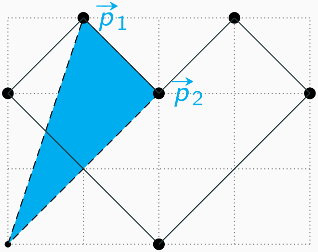

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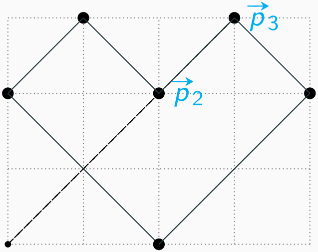

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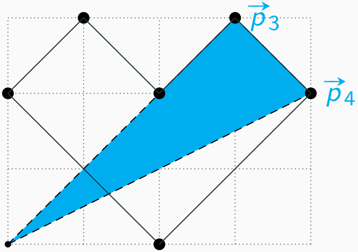

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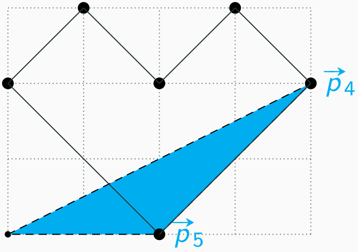

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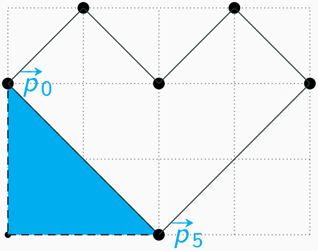
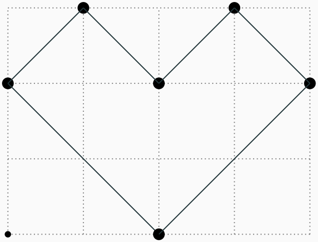


Optimising Area of Polygons

$$2A(\Omega) = \left| \sum_{i=0}^{n-1} \vec{p}_i \times \vec{p}_{i+1} \right|$$



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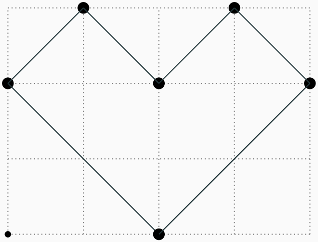

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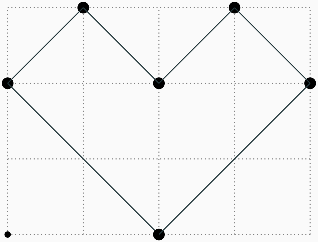

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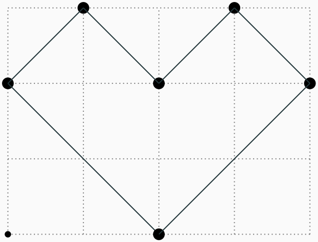

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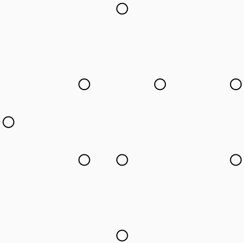
Constructing Convex Polygons - Graham Scan



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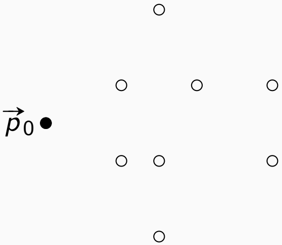
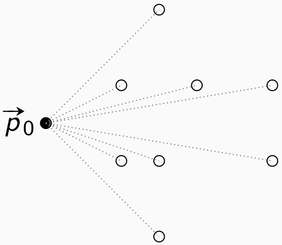


- pick a point $\vec{p}_0$ that is definitely on the convex hull

Constructing Convex Polygons - Graham Scan



- pick a point $\vec{p}_0$ that is definitely on the convex hull
- sort the remaining points according to their orientation from $\vec{p}_0$

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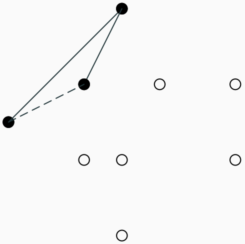

- pick a point $\vec{p}_0$ that is definitely on the convex hull
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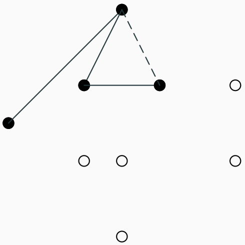

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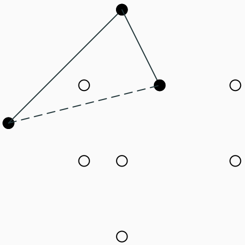

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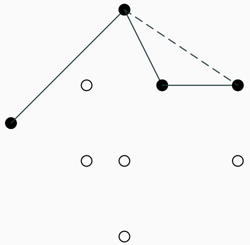

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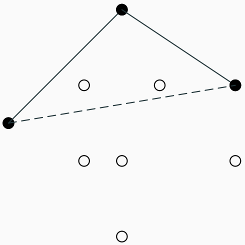

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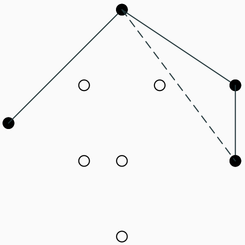

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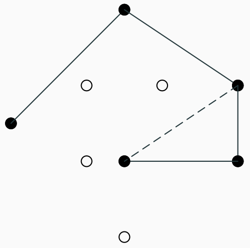

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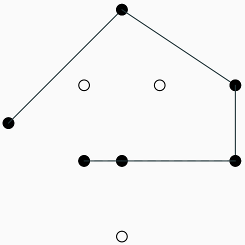

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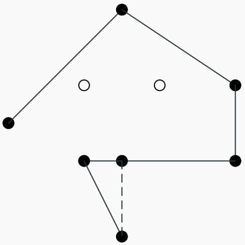

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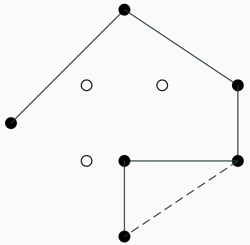

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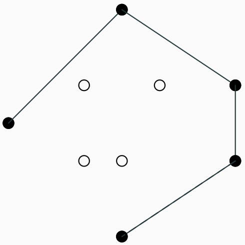

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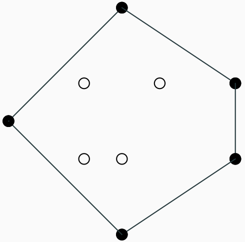

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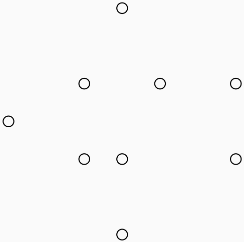
Constructing Convex Polygons - Jarvis' March



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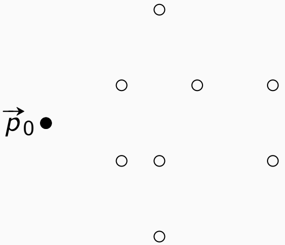

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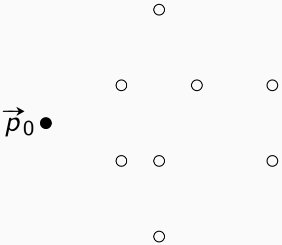

- pick a point $\vec{p}_0$ that is definitely on the convex hull
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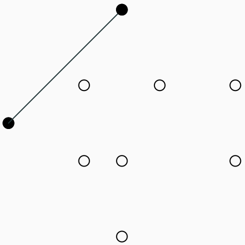

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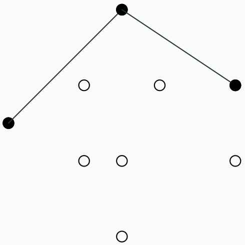

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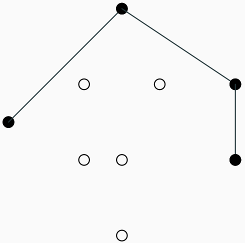

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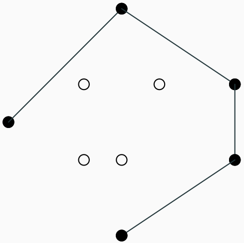

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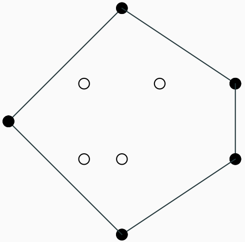

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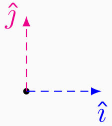
Linear Algebra

Basis Vectors

a vector lists directional components: x and y

- $\hat{\imath} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \hat{\jmath} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



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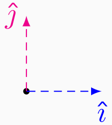

- $\hat{\imath} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\hat{\jmath} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- a vector is constructed by scaling and adding basis vectors
- $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow x\hat{\imath} + y\hat{\jmath}$

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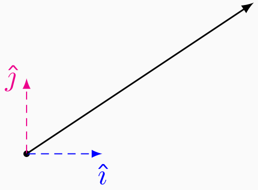

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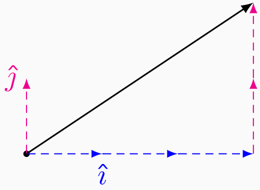

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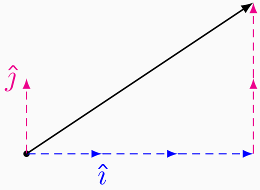

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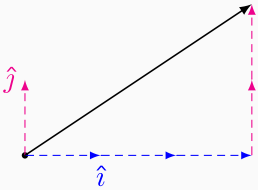
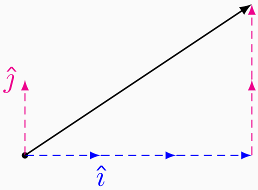
- $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow x\hat{\imath} + y\hat{\jmath}$
- $\hat{\imath}$ and $\hat{\jmath}$ can be grouped in a matrix $\begin{bmatrix} \hat{\imath} & \hat{\jmath} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Changing Basis Vectors



- by changing the basis vectors, any vector in a system can be changed

Changing Basis Vectors



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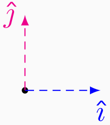
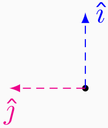


- by changing the basis vectors, any vector in a system can be changed
- rotation by 90 degrees

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Changing Basis Vectors



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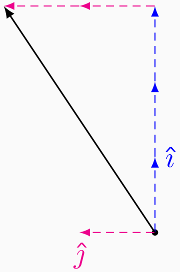

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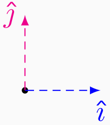

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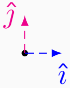
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- scaling

$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

Changing Basis Vectors



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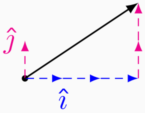

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Matrix Multiplication

these transformations are applied by multiplying a vector by a transformation matrix

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} ax + cy \\ bx + dy \end{bmatrix}$$